Gap solitons in a two-channel microresonator structure

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We show that, when two channel waveguides are coupled by a sequence of periodically spaced microresonators, the group-velocity dispersion is low in the vicinity of the gap associated with the resonant frequency of the resonators. This low dispersion permits the excitation of a gap soliton with much lower energy than in a gap of similar width caused by Bragg reflection.

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We consider nonlinear optical propagation in two channel waveguides coupled by periodically spaced microresonators [Fig. 1(a)]; we call such a device a two-channel, side-coupled, integrated, spaced sequence of resonators (SCISSOR).[1] The nonlinear properties of a similar structure with only one channel guide were studied previously,[1,2] as were the linear properties of the two-channel structure.[3,4] In a bottom (top) mode, light traveling in the forward direction in the bottom (top) channel is coupled via the resonator to light traveling in the backward direction in the top (bottom) channel. Two types of gap open in the dispersion relation: Bragg gaps associated with the resonator spacing, d, and resonator gaps associated with ρ, the radius of the resonators. We show that, for a Kerr nonlinear SCISSOR structure, the propagation of optical pulses is well described by a nonlinear Schrödinger equation (NLSE). The NLSE supports soliton solutions, and we find that much less energy is required for exciting a gap soliton in a resonator gap than in a Bragg gap with the same gap width.

We begin in the linear regime and denote the electric field in the bottom channel $\mathbf{L}(\mathbf{r}) = S(x,y)l(z)\hat{y}$, in the top channel $\mathbf{U}(\mathbf{r}) = S(x,y)u(z)\hat{y}$, and in the microresonator $\mathbf{Q}(y,R,\theta) = T(y,R)q(\theta)\hat{y}$, where $S(x,y)$ $[T(y,R)]$ is the mode profile associated with the channel waveguides (resonator waveguides), R is the radial variable, and θ is the angle within the resonator, measured counterclockwise from the bottom coupling points [see Fig. 1(b)]. We consider only the largest Cartesian component of the electric field in the channel and resonator, and to make the nonlinear term in the equation more tractable we take it to be the y component.

Away from the coupling points, the effect of propagation is the accumulation of phase by means of propagation constant $\nu = n_{\mathrm{eff}}\omega/c$, where ω is the frequency of the light and n_{eff} is the effective index of refraction associated with the waveguide; we ignore any small frequency dependence of n_{eff}. Without loss of generality, we assume that light is traveling forward (backward) in the bottom (top) channel. At the coupling points we use the model[1]

$$\begin{bmatrix} q(0_+) \\ l(a_+) \end{bmatrix} = \begin{bmatrix} \sigma_b & i\kappa_b \\ i\kappa_b & \sigma_b \end{bmatrix}\begin{bmatrix} q(0_-) \\ l(a_-) \end{bmatrix}, \tag{1}$$

where $a_\pm = a \pm (\delta a)$ and $0_\pm = \pm\delta\theta$, where δa and $\delta\theta$ are infinitesimal quantities. A similar expression is used for the top channel coupling point. To conserve energy, the coupling coefficients, σ_b, σ_t, κ_b, and κ_t, satisfy $|\sigma_i|^2 + |\kappa_i|^2 = 1$ and $\sigma_i^*\kappa_i = \sigma_i\kappa_i^*$, where $i = b, t$. Combining the effects of phase accumulation with those of coupling [Eq. (1)], we determine an expression that relates $l(d)$ and $u(d)$ to $l(0)$ and $u(0)$.

Searching for the Bloch solution, we write $l(d) = \exp(ikd)l(0)$ and $u(d) = \exp(ikd)u(0)$, where k is the Bloch wave number. Tracing the fields through the system, we find that

$$\begin{bmatrix} \exp(i\nu d)(\beta_b\beta_t - \alpha^2) - \beta_t\exp(ikd) & \alpha \\ -\alpha & \exp(-i\nu d) - \beta_t\exp(ikd) \end{bmatrix}\begin{bmatrix} l(0) \\ u(0) \end{bmatrix} = 0\,, \tag{2}$$

where $\alpha = i\gamma\kappa_b\kappa_t\exp(i\pi\nu\rho)$, $\beta_i = [\sigma_i + i\gamma\sigma_i\kappa_i \times \exp(2i\pi\nu\rho)]$, and $\gamma = i[1 - \sigma_b\sigma_t\exp(2i\pi\nu\rho)]^{-1}$. Equation (2) has nontrivial solutions only when the determinant of the matrix vanishes, from which we find an expression for the wave number, $k(\omega)$, that we can invert to determine the dispersion relation, $\omega(k)$.

We define the Bragg frequency, $\omega_b/c = \pi/(n_{\mathrm{eff}}d)$, and the resonator frequency, $\omega_r/c = 1/(n_{\mathrm{eff}}\rho)$. In Fig. 2 we plot the dispersion relation in the reduced band picture for a symmetric, two-channel SCISSOR structure with $n_{\mathrm{eff}} = 3.47$, $\sigma_b = \sigma_t = 0.98$, $2\pi\rho = 26\ \mu\mathrm{m}$, and $d = 16\ \mu\mathrm{m}$. There are two types of gap: the 72nd-order Bragg gap at $\omega/c \simeq 4.075\ \mu\mathrm{m}^{-1}$ and the 59th-order resonator gap at $\omega/c \simeq 4.11\ \mu\mathrm{m}^{-1}$. The upper and lower edges of the photonic bandgap occur at $k = 0$, whereas for the resonator gap they occur at $k = 0$, π/d. In the vicinity of a Bragg gap the

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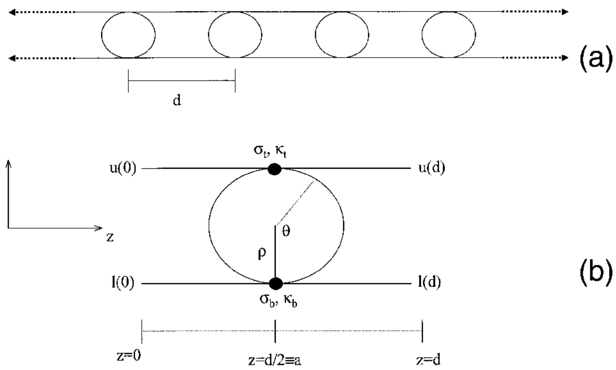

Fig. 1. (a) Schematic of the two-channel SCISSOR. (b) One unit cell of the structure. Filled circles, coupling points at the top and the bottom of the microresonator.

curvature of the dispersion relation is high, whereas near a resonator gap the bands are almost completely flat.

We now derive the NLSE that is relevant to the two-channel SCISSOR structure. We require the Bloch functions, $\mathbf{E}_{mk}(\mathbf{r})$, of the electric field,[5] where m is the index of the band. We can determine these functions by using the eigenvectors of Eq. (2) to find the electric field everywhere within one unit cell and then normalizing the field according to

$$\int \frac{\mathrm{d}\mathbf{r}}{\Omega_{\text{cell}}} n^2(\mathbf{r})\mathbf{E}_{mk}{}^*(\mathbf{r}) \cdot \mathbf{E}_{m'k'}(\mathbf{r}) = \delta_{mn'}\delta_{kk'}\,, \tag{3}$$

where Ω_{cell} is a normalization volume associated with one unit cell of the periodic medium. We assume that light is propagating in either a bottom or a top mode but not in both. We label the carrier wave number of the light $\bar{k}$, which corresponds to a frequency $\bar{\omega} = \omega(\bar{k})$, and introduce a field, $g_{m\bar{k}}(z, t)$, that is related to the energy in the electromagnetic field to lowest order through

$$\epsilon = \int |g_{m\bar{k}}(z,t)|^2 \mathrm{d}z\,. \tag{4}$$

The NLSE is

$$\left(i\frac{\partial}{\partial t} + i\omega_{m\bar{k}}{}'\frac{\partial}{\partial z}\right) g_{m\bar{k}}(z,t) = -\frac{1}{2}\,\omega_{m\bar{k}}{}''\,\frac{\partial^2 g_{m\bar{k}}(z,t)}{\partial z^2} - \Gamma_{m\bar{k}}|g_{m\bar{k}}(z,t)|^2 g_{m\bar{k}}(z,t)\,, \tag{5}$$

where $\omega_{m\bar{k}}{}' = \partial\omega/\partial k]_{\bar{k}}$ is the group velocity at the carrier wave number and $\omega_{m\bar{k}}{}'' = \partial^2\omega/\partial k^2]_{\bar{k}}$ is the group-velocity dispersion. The nonlinear coefficient is given by

$$\Gamma_{m\bar{k}} = \frac{3\bar{\omega}}{4A_{\text{eff}\ \epsilon_0}} \int_{\text{cell}} \frac{\mathrm{d}\mathbf{r}}{\Omega_{\text{cell}}}\,\chi^{(3)}(\mathbf{r})\,|\mathbf{E}_{m\bar{k}}(\mathbf{r})|^4, \tag{6}$$

where $\chi^{(3)}$ ($\mathbf{r}$) is the nonlinear susceptibility of the medium and is assumed to have the same periodicity as the device and A_{eff} is an effective area associated with the cross section of the channel waveguides. The region of validity of Eq. (5) has been extensively discussed.[5,6]

When the carrier frequency of the field is at an upper band edge, the NLSE [Eq. (5)] supports gap soliton solutions.[7] We set $m = u$ to represent the upper band. The solitons have the form[7]

$$g_{u\bar{k}}(z,t) = A\exp(iB_2 z)\exp[-i(\delta + \Delta)t]\mathrm{sech}(B_1 z - Ct)\,, \tag{7}$$

with $A = (-2\delta/\Gamma_{u\bar{k}})^{1/2}$, $B_1 = (-2\delta/\omega_{u\bar{k}}{}'')^{1/2}$, $B_2 = (+2\Delta/\omega_{u\bar{k}}{}'')^{1/2}$, and $C = \omega_{u\bar{k}}{}''B_1B_2$, where $\omega_{u\bar{k}}{}''$ is the group-velocity dispersion at the upper band edge and where the signs of the detunings δ and Δ are chosen such that these coefficients come out to be real; δ determines the height and the spatial width of the soliton, whereas Δ determines the velocity. The center frequency of the soliton is $\omega_c = \omega_{u\bar{k}} + \delta + \Delta$, and the frequency width of the pulse is denoted C. For most of the frequencies of the pulse to be contained within the gap we require that $\omega_c + 2C \le \omega_{u\bar{k}}$. It is easy to confirm that this condition can be met for an arbitrary value of C if we set $\Delta = C/(2M)$ and $\delta = -C(M/2)$, where $M \ge 4$. However, the pulse width is limited by the fact that NLSE (5) is valid only for frequencies slightly inside the gap. If we fix $M = 4$ then we should have $C(\overline{\delta\omega})/20$, where $\overline{\delta\omega}$ is the width of the gap. We have verified that a pulse of this width and central frequency is well described by the NLSE.

Using the form of $g_{u\bar{k}}$ [Eq. (7)] in the expression for the energy [Eq. (4)], we find that $\epsilon_{u\bar{k}}{}^{\text{soliton}} = (2\sqrt{2|\delta|}/\Gamma_{u\bar{k}})\sqrt{\omega_{u\bar{k}}{}''}$. Because the group-velocity dispersion near a resonator gap is so much smaller than near a Bragg gap, the energy required for exciting a gap soliton with the same pulse width (C), and the same depth within the gap ($\delta + \Delta$), is much lower in a resonator gap; furthermore, the resonator soliton will travel with a slower group velocity and will

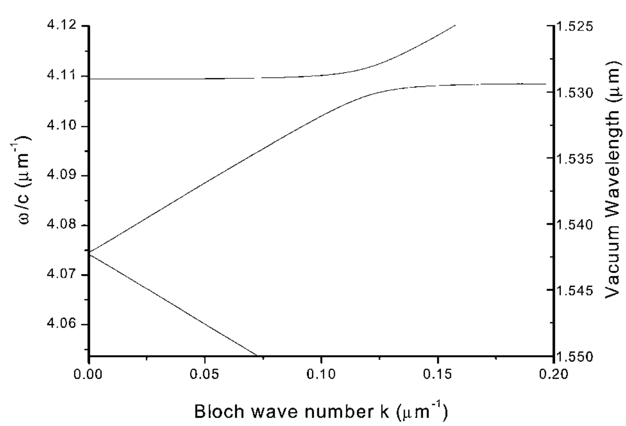

Fig. 2. Dispersion relation for the two-channel SCISSOR with material parameters given in the text. The gap at $\omega/c = 4.11\ \mu\text{m}^{-1}$ is associated with the 59th-order resonance of the microresonator. The gap at $\omega/c = 4.075\ \mu\text{m}^{-1}$ is associated with Bragg reflection. These gaps are at typical communications wavelengths, as indicated by the right-hand axis.

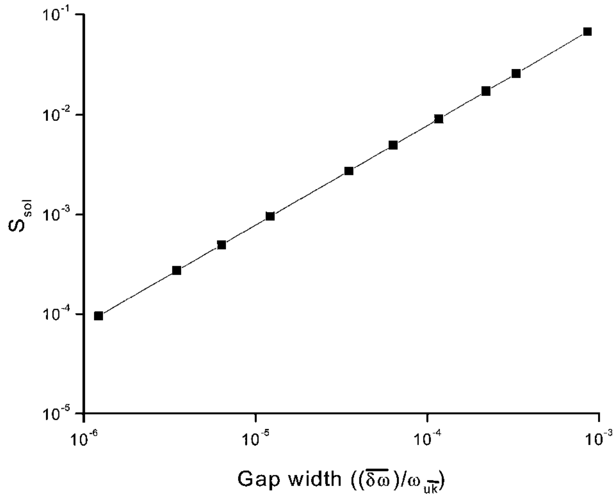

Fig. 3. $S_{\rm sol}$ is the ratio of the energy required for forming a gap soliton in a resonator gap to the energy required for forming the same gap soliton in a Bragg gap with the same gap width relative to its center frequency.

consequently have a smaller spatial width. We define a quantity $S_{\rm sol}(\overline{\delta\omega}) = \epsilon_{m\bar{k}}{}^{\rm soliton}]_{\rm res}/\epsilon_{m\bar{k}}{}^{\rm soliton}]_{\rm Bragg}$, where $\epsilon_{m\bar{k}}{}^{\rm soliton}]_{\rm res(Bragg)}$ is the energy required for exciting a gap soliton in a resonator (Bragg) gap; $S_{\rm sol}$ is a measure of how much easier it is to form a gap soliton in a resonator gap than in a Bragg gap. To make this comparison we consider one system in which $\omega_{u\bar{k}}$ corresponds to the upper band edge of a resonator gap and another in which the same frequency $\omega_{u\bar{k}}$ corresponds to the upper band edge of a Bragg gap. The overlap integrals that we use to determine the nonlinear coefficient, $\Gamma_{m\bar{k}}$, are roughly equal at the gaps, so $S_{\rm sol} \simeq (\omega_{\rm res}''/\omega_{\rm Bragg}'')^{1/2}$. We use physical parameters defined above but vary the values of σ and d to achieve different gap widths and center frequencies.

In Fig. 3 we plot the value of $S_{\rm sol}$ as a function of the gap width $(\overline{\delta\omega}/\omega_{u\bar{k}})$. For a small gap width, $(\overline{\delta\omega}/\omega_{u\bar{k}}) = 10^{-6}$, $S_{\rm sol} = 10^{-4}$; for gap width $(\overline{\delta\omega}/\omega_{u\bar{k}}) = 10^{-4}$, which is more realistic, $S_{\rm sol} = 10^{-2}$. Of course, material and mode dispersion, both neglected in our calculations, will set a lower bound on $S_{\rm sol}$. The low energy requirements for gap solitons in a resonator gap are balanced by a much longer soliton formation length,[8] but for switching applications this restriction is not so important. A pulse with a form similar to Eq. (7) but with a much lower amplitude will be unable to propagate, because all its frequencies lie within the gap. By contrast, if the pulse has the correct amplitude, it will form into a soliton while it propagates. If the initial pulse is close to a soliton, then reshaping should be minimal.

In conclusion, we have investigated optical propagation in a two-channel SCISSOR structure with a weak Kerr nonlinearity. We have presented a NLSE that accurately describes propagation near the band edges of a resonator gap if the light is propagating in only one mode of the system. The energy required for forming a gap soliton is much smaller than in a Bragg gap of similar width. We note, too, that whereas the one-channel SCISSOR structure investigated by Heebner *et al.*[1] supports solitons that can travel with a small group velocity, that velocity can never vanish; furthermore, that structure possesses no gap, so a true gap soliton could not be launched. We intend to extend the analysis to coupled gap solitons and to discuss the issues involved in experimentally launching and observing gap solitons.

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